Worcester County Mathematics League
Junior Varsity Meet 1 - October 30, 2024
Round 1 - Order of Operations

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Evaluate

$$7 + 6 \div 2 \times (3 + 1) - 5$$

2. Evaluate the expression for $a = 3$ and $b = a^a - 25$.

$$a^b + b^a + (a + b)^{a-b}$$

3. For nonnegative integers a, b, we define

$$a \oplus b = \begin{cases} a, & \text{if } b = 0 \\ (a \oplus (b - 1)) + a, & \text{if } b > 0 \end{cases}$$

Compute $7 \oplus 9$.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

All answers must be in simplest exact form in the answer section.

NO CALCULATORS ALLOWED

1. Solve for k.

$$3k + 7 = 8k - 23$$

2. Solve for x.

$$\frac{2x - 5}{5} + \frac{1}{20}(x + 8) = -x + \frac{1}{4}$$

3. Solve for all values of a.

$$(2a - 7)^{2a+6} = 1$$

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Alice has 2 nickels and 3 pennies. How many different total nonnegative values can she create using any combination of these coins?

2. In a 2 by 2 grid, a one-digit number is placed in each cell. The product of all four numbers is 70, and the two numbers in the top row have a sum of 7. What is the sum of the two numbers in the bottom row?

3. Four people—Alex, Blake, Casey, and Drew—are talking about two variables a and b, which are positive integers. Each of them makes the following statements:

   - **Alex:** "Either $a = 3$ or $b = 5$, but not both."
   - **Blake:** "If $a \neq 3$, then $b = 4$."
   - **Casey:** "Both a and b are greater than 4."
   - **Drew:** "Either $a = 5$ or $b = 3$, or both."

   Exactly one of these four is lying. For each possible tuple (a, b), evaluate the product ab, then find the sum of all these products.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. If six cows cost 56 dollars, how much would fifteen cows cost?

2. A bag contains only red, green, and blue marbles. The ratio of red to green marbles is $2 : 3$. The ratio of the total number of red and green marbles to the number of blue marbles is $10 : 11$. What fraction of the all of the marbles are green?

3. Thirteen pirates are dividing their gold in a peculiar way. The first pirate divides the gold into five equal piles and takes one pile. The second pirate then divides each of the remaining four piles into six equal piles and takes one pile from each group of six piles. In general, the nth pirate divides each remaining pile into $n + 4$ equal piles and takes one pile from each group of $n + 4$ piles. What fraction of the total gold remains after the 13th pirate has taken their share?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Compute 1 - 2 + 4 - 8 + 16 - 32 + 64.

2. Solve for t.

$$t + 2t + 3t - 4 - 5 - 6 = 1 + 2 + 3 - 4t - 5t - 6t$$

3. If one pen is worth three pencils and one marker is worth seven pens, how many pencils are equivalent to 11 markers and 8 pens?

4. How many pairs of 1-digit numbers, x and y satisfy $xy > 30$?

5. There are two overlapping squares. One square has a side length of 6, and the second square has a side that is the diagonal of the first square. Find the total area occupied by both squares, taking into account their overlap?

6. How many pairs of positive integers a and b satisfy

$$2a + 3b = 50$$

7. What is the smallest positive integer that is divisible by 7 and has a digit sum of 4?

8. Selena draws an square. Then, for each vertex, she draws line segments to the midpoints of the two sides it is not adjacent to. How many regions is Selena's square split into?

ANSWERS

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

Worcester County Mathematics League
Junior Varsity Meet 1 - October 30, 2024
Answer Key

Round 1 - Evaluation and Order of Operations

1. 14

2. 22

3. 70

Round 2 - Linear Equations

1. 6

2. $\frac{17}{29}$

3. $-3, 3, 4$ (in any order)

Round 3 - Fractions...

1. 12

2. 8

3. 34

Round 4 - Ratio, Proportion, and Variation

1. 140

2. $\frac{2}{7}$

3. $\frac{4}{17}$

Team Round

1. 43

2. 1

3. 255

4. 26

5. 90

6. 8

7. 112

8. 25

School: _____

Circle your team below.

Team #1   Team #2   Team #3   Team #4   Team #5

Team #6   Team #7   Team #8   Team #9   Team #10

Name: _____

For Official Use Only
Score:

YES   Western Mass ARML Member?   NO
(circle one)

Reminders:

- No two students from the same school should be sitting at the same table or in adjacent desks in a classroom.

- All electronic devices should be OFF and not in plain sight.

- Calculators are **not** allowed during this round.

- Do not turn this paper over until the moderator says, "Begin!"

WOCOMAL Junior Varsity Meet 1
Round 1: Order of Operations

School: _____

Circle your team below.

Team #1   Team #2   Team #3   Team #4   Team #5

Team #6   Team #7   Team #8   Team #9   Team #10

Name: _____

YES   Western Mass ARML Member?   NO
(circle one)

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WOCOMAL Junior Varsity Meet 1
Round 2: Linear Equations

School: _____

Name: _____

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- Calculators are **not** allowed during this round.

- Do not turn this paper over until the moderator says, "Begin!"

WOCOMAL Junior Varsity Meet 1
Round 3: Logic Problems

WOCOMAL Junior Varsity Meet 1
Round 4: Ratio, Proportion, and Variation

Worcester County Mathematics League

Junior Varsity Meet 1 - October 30, 2024

TEAM ROUND

School Name: _____

Team #: _____

Team Members:

_____

_____

_____

_____

_____

Score = _____ / 16

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Evaluate

$$7 + 6 \div 2 \times (3 + 1) - 5$$

Solution:
$$= 7 + 6 \div 2 \times 4 - 5$$
$$= 7 + 3 \times 4 - 5$$
$$= 7 + 12 - 5$$
$$= 19 - 5 = \boxed{14}$$

2. Evaluate the expression for $a = 3$ and $b = a^a - 25$.

$$a^b + b^a + (a + b)^{a-b}$$

Solution: We have $b = 3^3 - 25 = 2$. Then,

$$a^b + b^a + (a + b)^{a-b} = 3^2 + 2^3 + (3 + 2)^{3-2}$$
$$= 9 + 8 + 5^1 = \boxed{22}$$

3. For nonnegative integers a, b, we define

$$a \oplus b = \begin{cases} a, & \text{if } b = 0 \\ (a \oplus (b-1)) + a, & \text{if } b > 0 \end{cases}$$

Compute $7 \oplus 9$.

Solution: We have
$$7 \oplus 9$$
$$= 7 \oplus 8 + 7 = 7 \oplus 8 + 1 \cdot 7$$
$$7 \oplus 7 + 7 + 1 \cdot 7 = 7 \oplus 7 + 2 \cdot 7$$
$$\dots$$
$$7 \oplus 0 + 9 \cdot 7 = 7 + 63 = \boxed{70}$$

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Solve for k.

$$3k + 7 = 8k - 23$$

Solution:
$$\Rightarrow 30 = 5k \Rightarrow k = \boxed{6}$$

2. Solve for x. Give your answer as a fraction.

$$\frac{2x - 5}{5} + \frac{1}{20}(x + 8) = -x + \frac{1}{4}$$

Solution: Multiplying by 20 on both sides, we get

$$4(2x - 5) + (x + 8) = -20x + 5$$
$$\Rightarrow 8x - 20 + x + 8 = -20x + 5$$
$$\Rightarrow 9x - 12 = -20x + 5$$
$$\Rightarrow 29x = 17$$
$$\Rightarrow x = \boxed{\frac{17}{29}}$$

3. Solve for all values of a.

$$(2a - 7)^{2a+6} = 1$$

Solution: If $2a - 7 \neq \pm 1$, we see that $2a + 6$ must be 0. Solving, we get $a = -3$. Otherwise, we need $2a - 7 = \pm 1$, giving potential solutions $a = 3, 4$. Plugging in these potential solutions, we get that they all work, so all solutions are $a = \boxed{-3, 3, 4}$.

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Alice has 2 nickels and 3 pennies. How many different total nonnegative values can she create using any combination of these coins?

> **Solution:** She can choose to use 0, 1, or 2 nickels and 0, 1, 2, or 3 pennies, with each combination of nickels and pennies giving a different value. Thus, there are $3 \cdot 4 = \boxed{12}$ different values she can create.

2. In a 2 by 2 grid, a one-digit number is placed in each cell. The product of all four numbers is 70, and the two numbers in the top row have a sum of 7. What is the sum of the two numbers in the bottom row?

> **Solution:** Since 5 and 7 are factors of 70 and are the only positive one-digit numbers divisible by 5 and 7, respectively, they must be among the four numbers. This means that the remaining numbers multiply to 2, so the remaining two numbers must be 1 and 2. For the sum of the top row to be 7, the numbers must be 2 and 5. Thus, 1 and 7 are in the bottom row, giving a sum of $\boxed{8}$.

3. Four people—Alex, Blake, Casey, and Drew—are talking about two variables a and b, which are positive integers. Each of them makes the following statements:

   - **Alex:** "Either $a = 3$ or $b = 5$, but not both."
   - **Blake:** "If $a \neq 3$, then $b = 4$."
   - **Casey:** "Both a and b are greater than 4."
   - **Drew:** "Either $a = 5$ or $b = 3$, or both."

   Exactly one of these four is lying. For each possible tuple (a, b), evaluate the product ab, then find the sum of all these products.

Solution: If Alex and Drew are both telling the truth, we see that $(a, b) = (3, 3)$ or $(a, b) = (5, 5)$. Both of these work. Otherwise, exactly one of them is telling the truth and the other is lying. If Alex is the truthful one and $a = 3$, Casey's statement cannot be true. If Alex is the truthful one and $b = 5$, by Casey's statement, $a > 4$, so by Blake's statement, $b = 4$, contradiction. If Drew is the truthful one and $a = 5$, b must be 4 by Blake's statement, which contradicts Casey. If Drew is the truthful one and $b = 3$, Casey's statement is contradicted. Thus, $(3, 3)$ and $(5, 5)$ are the only solutions, giving an answer of $\boxed{34}$.

All answers must be in simplest exact form in the answer section.

NO CALCULATORS ALLOWED

1. If six cows cost 56 dollars, how much would fifteen cows cost?

Solution: Fifteen cows would cost

$$\frac{15 \cdot 56}{6} = \frac{5 \cdot 56}{2} = 5 \cdot 28 = \boxed{140}$$

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2. A bag contains only red, green, and blue marbles. The ratio of red to green marbles is $2 : 3$. The ratio of the total number of red and green marbles to the number of blue marbles is $10 : 11$. What fraction of the all of the marbles are green?

Solution: We have $r = \frac{2}{3}g$. Also, we have

$$\frac{r + g}{b} = \frac{10}{11}$$

$$\Rightarrow \frac{\frac{2}{3}g + g}{b} = \frac{10}{11}$$

$$\Rightarrow \frac{\frac{5}{3}g}{b} = \frac{10}{11}$$

$$\Rightarrow b = \frac{11}{6}g$$

Thus,

$$\frac{g}{r + g + b} = \frac{g}{\frac{2}{3}g + g + \frac{11}{6}g} = \frac{g}{\frac{21}{6}g} = \boxed{\frac{2}{7}}$$

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3. Thirteen pirates are dividing their gold in a peculiar way. The first pirate divides the gold into five equal piles and takes one pile. The second pirate then divides each of the remaining four piles into six equal piles and takes one pile from each group of six piles. In general, the nth pirate divides each remaining pile into $n + 4$ equal piles and takes one pile from each group of $n + 4$ piles. What fraction of the total gold remains after the 13th pirate has taken their share?

Solution: We see that the n pirate leaves behind $\frac{n+3}{n+4}$ of the remaining gold. Thus, the fraction of gold remaining at the end is

$$\frac{4}{5}\frac{5}{6}\cdots\frac{16}{17} = \boxed{\frac{4}{17}}$$

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Compute 1 - 2 + 4 - 8 + 16 - 32 + 64.

Solution:
$$= \boxed{43}$$

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2. Solve for t.

$$t + 2t + 3t - 4 - 5 - 6 = 1 + 2 + 3 - 4t - 5t - 6t$$

Solution:
$$\Rightarrow 21t = 21 \Rightarrow t = \boxed{1}$$

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3. If one pen is worth three pencils and one marker is worth seven pens, how many pencils are equivalent to 11 markers and 8 pens?

Solution: One marker is worth $7 \cdot 3 = 21$ pencils. Thus, 11 markers and 8 pens are worth $11 \cdot 21 + 3 \cdot 8 = 231 + 24 = \boxed{255}$ pencils.

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4. How many pairs of 1-digit numbers, x and y satisfy $xy > 30$?

Solution: If $x \le 3$, there are no solutions. If $x = 4$, $y \ge 8$, giving 2 solutions. If $x = 5$, $y \ge 7$, giving 3 solutions. If $x = 6$, $y \ge 6$, giving 4 solutions. If $x = 7$, $y \ge 5$, giving 5 solutions. If $x = 8$, $y \ge 4$, giving 6 solutions. If $x = 9$, $y \ge 4$, giving 6 solutions. This gives a total of $2 + 3 + 4 + 5 + 6 + 6 = \boxed{26}$ solutions.

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5. There are two overlapping squares. One square has a side length of 6, and the second square has a side that is the diagonal of the first square. Find the total area occupied by both squares, taking into account their overlap?

> **Solution:** The second square has a side lenght of $6\sqrt{2}$, with area $6^2 \cdot 2 = 72$. The first square has area $6^2 = 36$. Their intersection is half of the first square, so it has area 18. Thus the total area is $72 + 36 - 18 = \boxed{90}$.

6. How many pairs of positive integers a and b satisfy

$$2a + 3b = 50$$

> **Solution:** The solutions are $(1, 16), (4, 14), \ldots, (22, 2)$, giving $\boxed{8}$ solutions.

7. What is the smallest positive integer that is divisible by 7 and has a digit sum of 4?

> **Solution:** Running through multiples of 7, we get that the answer is $\boxed{112}$.

8. Selena draws an square. Then, for each vertex, she draws line segments to the midpoints of the two sides it is not adjacent to. How many regions is Selena's square split into?

> **Solution:** Drawing it out, we can count $\boxed{25}$ regions.

ANSWERS

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

Round 1 - Order of Operations

1. 14
2. 22
3. 70

Round 2 - Linear Equations

1. 6
2. $\frac{17}{29}$
3. $-3, 3, 4$ (in any order)

Round 3 - Fractions...

1. 12
2. 8
3. 34

Round 4 - Ratio, Proportion, and Variation

1. 140
2. $\frac{2}{7}$
3. $\frac{4}{17}$

Team Round

1. 43
2. 1
3. 255
4. 26
5. 90
6. 8
7. 112
8. 25

School: _____

Circle your team below.

Team #1   Team #2   Team #3   Team #4   Team #5

Team #6   Team #7   Team #8   Team #9   Team #10

Name: _____

For Official Use Only
Score:

YES   **Western Mass ARML Member?**   NO
(circle one)

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- Do not turn this paper over until the moderator says, "Begin!"

WOCOMAL Junior Varsity Meet 3
Round 1: Order of Operations

School: _____

Name: _____

Reminders:

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- All electronic devices should be OFF and not in plain sight.

- Calculators are **not** allowed during this round.

- Do not turn this paper over until the moderator says, "Begin!"

WOCOMAL Junior Varsity Meet 3
Round 2: Linear Equations

School: _____

Circle your team below.

Team #1   Team #2   Team #3   Team #4   Team #5

Team #6   Team #7   Team #8   Team #9   Team #10

Name: _____

YES   Western Mass ARML Member?   NO
(circle one)

Reminders:

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WOCOMAL Junior Varsity Meet 3
Round 3: Logic Problems

School: ────────────────────────────────

Name: ────────────────────────────────

Reminders:

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- Do not turn this paper over until the moderator says, "Begin!"

WOCOMAL Junior Varsity Meet 3
Round 4: Ratio, Proportion, and Variation

Worcester County Mathematics League

Junior Varsity Meet 3 - March 27, 2024

TEAM ROUND

School Name: _____

Team #: _____

Team Members:

_____

_____

_____

_____

_____

Score = _____ / 16